CONVERGENCE OF A LANDEN TRANSFORMATION DEGREE SIX

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An integral $I(a,b)$ is said to be invariant with respect to the transformation $a_{n+1} = f(a_n, b_n)$, $b_{n+1} = g(a_n, b_n)$ if $I(a_{n+1}, b_{n+1}) = I(a_n, b_n)$. A *rational Landen transformation* is one that preserves the integral of a rational function over the positive real line. Boros and Moll have shown that the integral of a rational function of degree six is invariant under the transformation $a_{n+1} = (a_n b_n + 5a_n + 5b_n + 9)/(a_n + b_n + 2)^{4/3}$ and $b_{n+1} = (a_n + b_n + 6)/(a_n + b_n + 2)^{1/3}$. Furthermore, they have shown that the sequence $(a_{n+1},\ b_{n+1})$ converges to the point $(3, 3)$ if $a_0, b_0 > 0$. We show that the region of convergence for $(a_n,\ b_n)$ can be extended from the first quadrant in the ab-plane to include the area in the second quadrant above the parabola $a^2/3 = b$. The Boros-Moll conjecture states that the full region of convergence of the sequence $(a_n,\ b_n)$ corresponds to the region of convergence for the integral $I(a,b)$. We take a step towards the proof of this conjecture by showing that the integral converges precisely on the region bounded by the curve $4b^3 - a^2b^2 - 18ab + 4a^3 + 27 = 0$.