INTEGRAL CLOSURE OF MONOMIAL IDEALS: THE SQUARE CASE

Sotero Alvarado, University of California, San Diego; Angela Esquibel, University of New Mexico; Lianette Passapera-Sánchez, University of Puerto Rico-Humacao

In this project we study certain types of polynomial *ideals*, that is, certain collections of polynomials functions $f(x, y, z)$ in three variables x, y, z, that are closed under addition and multiplication by arbitrary polynomials. A set that captures the entire geometry associated with the ideal is its *radical*, a large ideal that may be difficult to study. A stepping stone to the radical is the ideal's *integral closure*, the set of all elements that are zeros of a specific polynomial equation. A natural question is to determine whether an ideal is equal to its integral closure; such ideals are called *integrally closed*. We investigate when the product of two integrally closed ideals is again integrally closed. In particular, we take the product of a monomial ideal I with itself, I^2, and try to determine whether it is integrally closed. We describe an infinite family of integrally closed ideals that when squared remain integrally closed. Furthermore, we produce a simple algorithm to determine whether I^2 is integrally closed.