BINOMIAL IDEALS FROM GRAPHS

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In this project we study relationships among the different *spanning trees* of a graph, that is, all the subgraphs that contain no cycles and contain all vertices of the graph. The spanning trees can be represented as the columns of a 0/1-matrix M, whose rows are labeled by the edges of the graph. Using M, one can define a homomorphism ϕ_M between the polynomial ring using the spanning trees S_i as variables and the polynomial ring on the edges e_j of the graph. This homomorphism maps each spanning tree to the product of its edges $S_i \mapsto e_1^{a1} \cdot e_2^{a2} \cdots e_m^{am}$. The *kernel* of this mapping, $\ker(\phi_M)$, is an ideal that encodes all relations among the spanning trees of the graph.

In our research we study several combinatorial conjectures that can be formulated as properties of special generating sets of $\ker(\phi_M)$. We present proofs of these conjectures for several infinite classes of graphs including planar graphs.